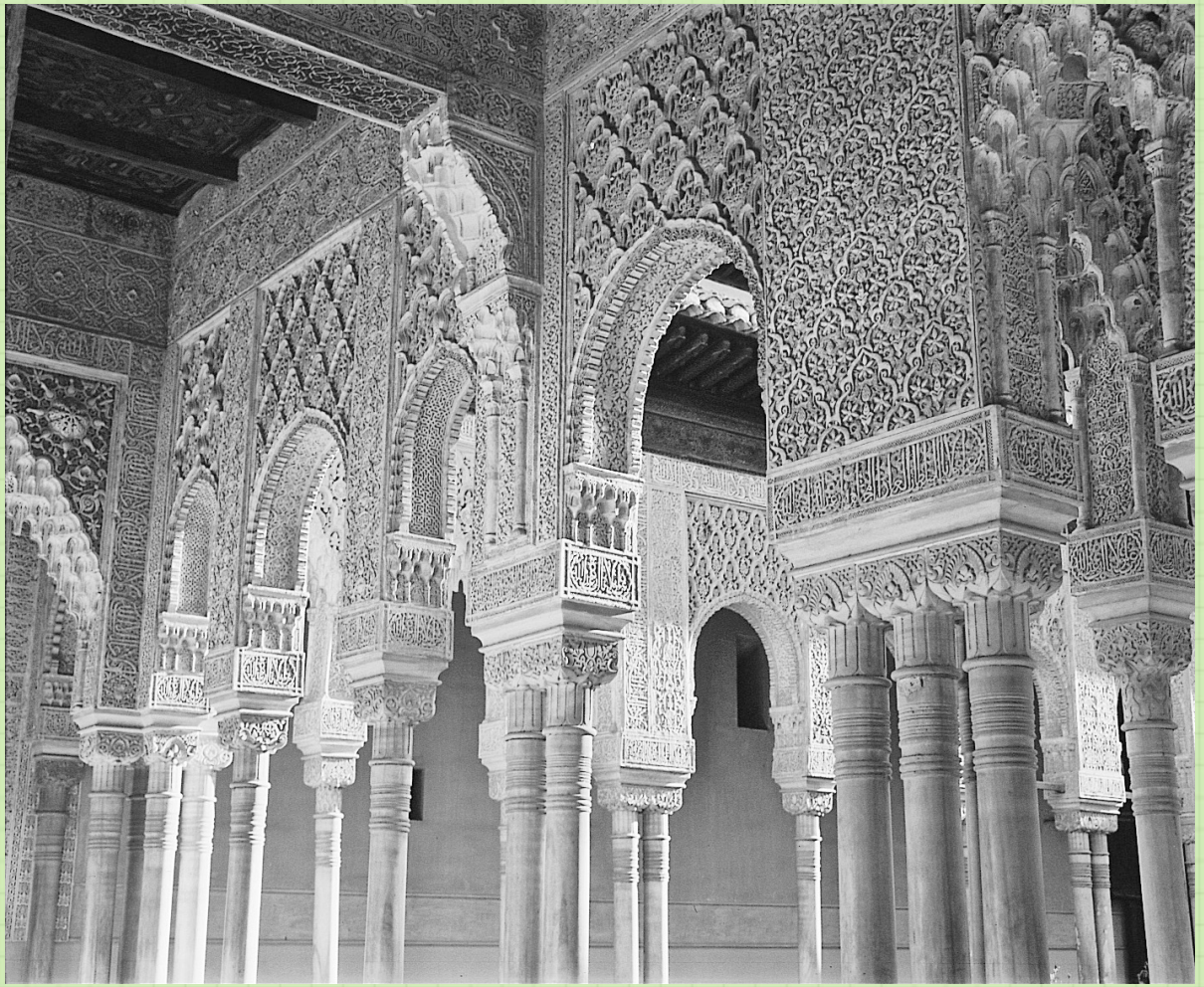


Chapter 9: Columns

And as imagination bodies forth the forms of things unknown,
The poet's pen turns them to shapes
And gives to airy nothingness a local habitation and a name.

William Shakespeare
A Midsummer Night's Dream

Equilibrium Regimes

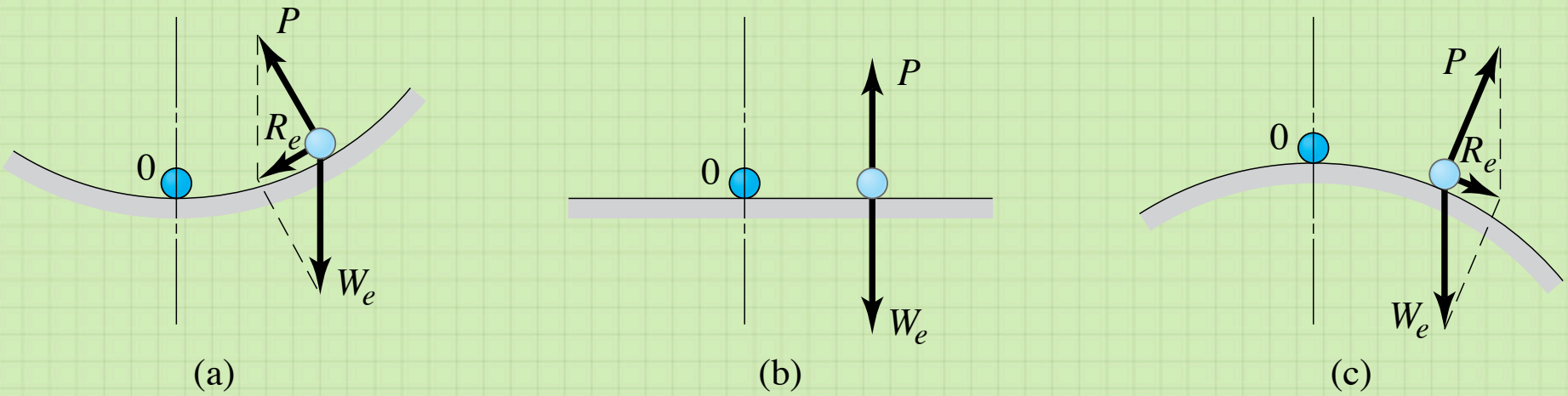


Figure 9.1 Depiction of equilibrium regimes. (a) Stable; (b) neutral; (c) unstable.

Example 9.1

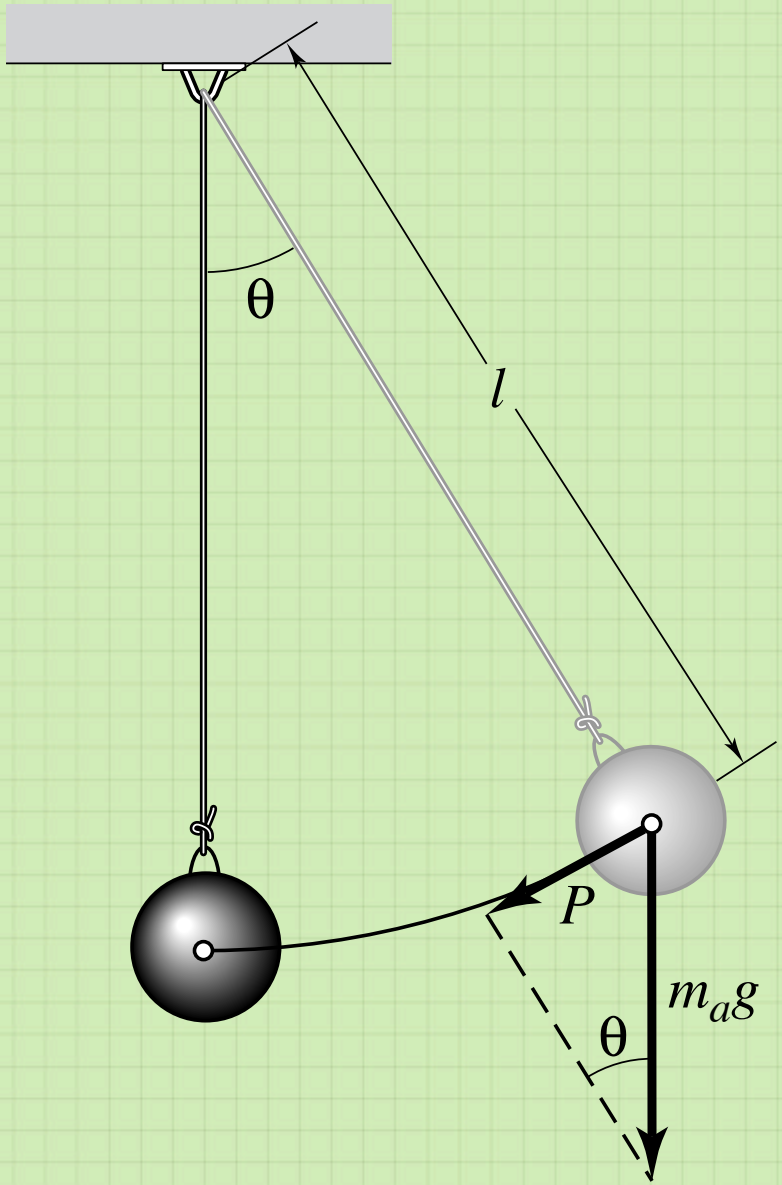


Figure 9.2 Pendulum used in Example 9.1.

Column with Pinned Ends

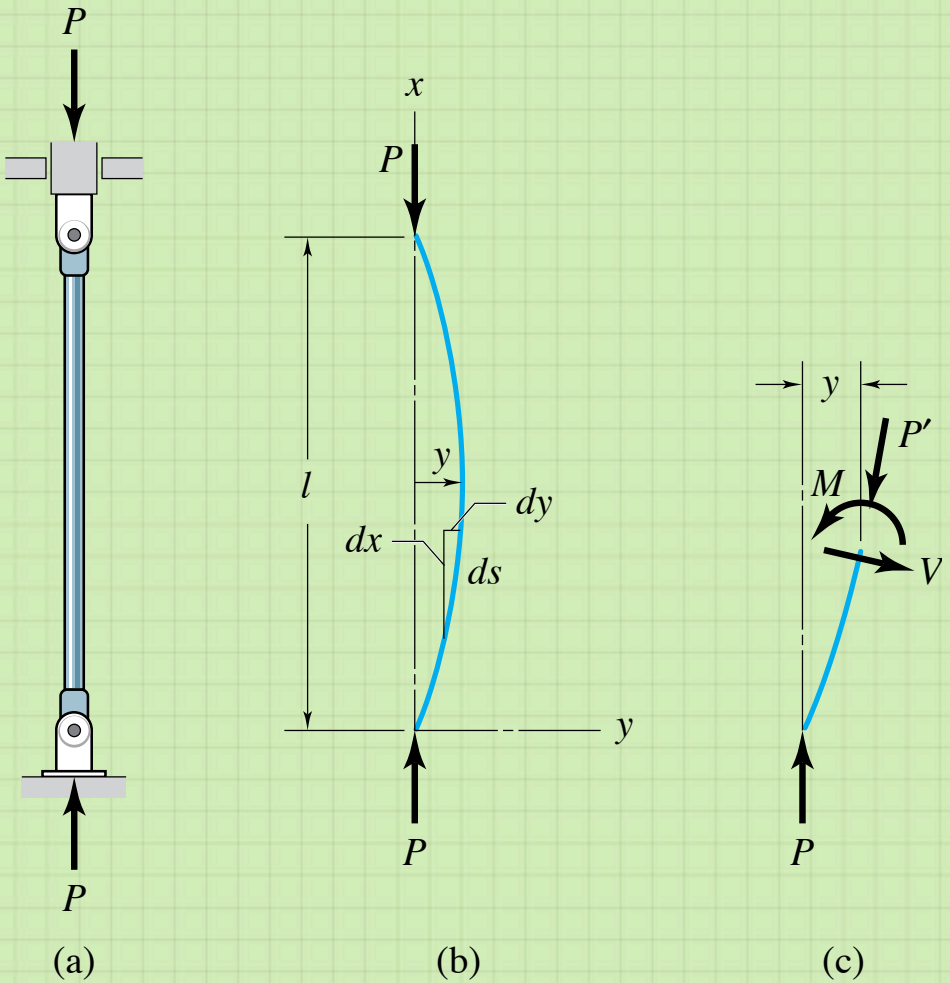


Figure 9.3 Column with pinned ends. (a) Assembly; (b) deformation shape; (c) load acting.

Buckling of Columns

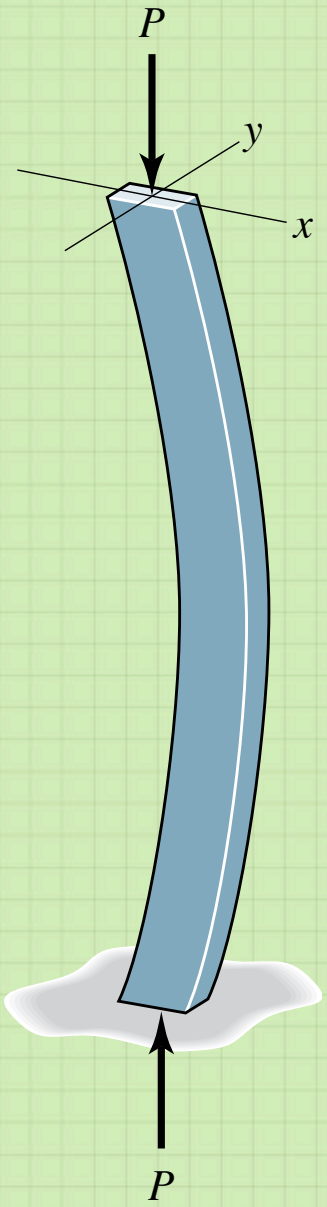


Figure 9.4 Buckling of rectangular section.

Euler Equation:

$$P_{cr} = \frac{n^2 \pi^2 EI}{l^2}$$

Column Effective Lengths

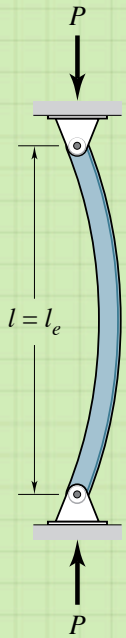
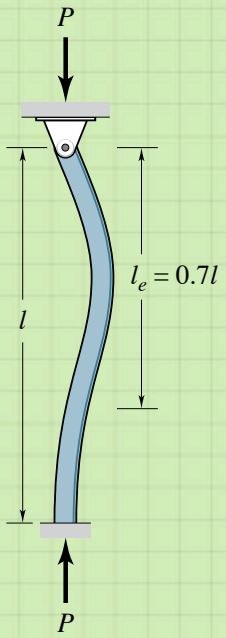
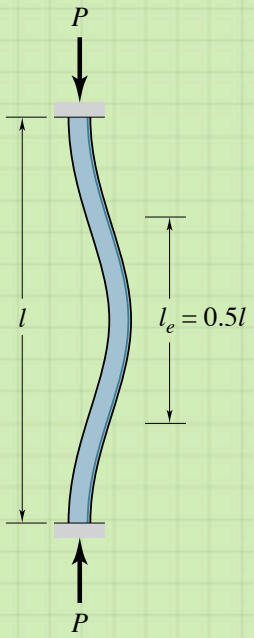
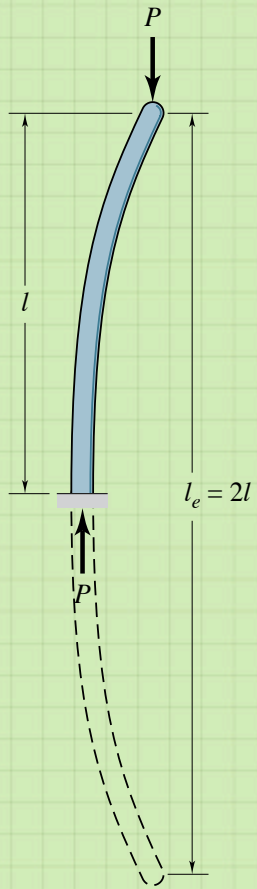
End condition description	Both ends pinned	One end pinned and one end fixed	Both ends fixed	One end fixed and one end free
Illustration of end condition				
Theoretical effective column length	$l_e = l$	$l_e = 0.7l$	$l_e = 0.5l$	$l_e = 2l$
AISC (1989)– recommended effective column length	$l_e = l$	$l_e = 0.8l$	$l_e = 0.65l$	$l_e = 2.1l$

Table 9.1 Effective length for four end conditions.

Buckling for Different Slenderness Ratio

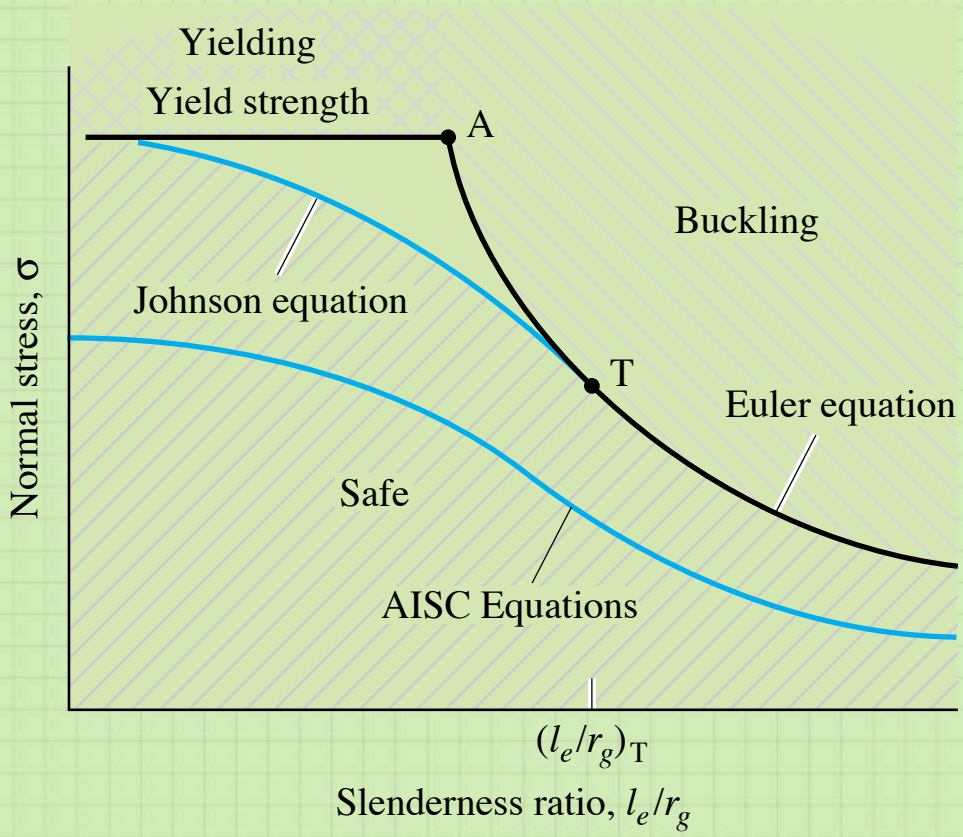


Figure 9.5 Normal stress as a function of slenderness ratio.

Critical Slenderness Ratio

$$C_c = \left(\frac{l_e}{r_g}\right)_E = \sqrt{\frac{2E\pi^2}{S_y}}$$

Euler Equation

$$(\sigma_{cr})_E = \frac{(P_{cr})_E}{A} = \frac{\pi^2 E}{(l_e/r_g)^2}$$

Johnson Parabola

$$(\sigma_{cr})_J = \frac{(P_{cr})_J}{A} = S_y - \frac{S_y^2}{4\pi^2 E}\left(\frac{l_e}{r_g}\right)^2$$

AISC Buckling Criterion

Elastic Buckling $$\sigma_{all} = \frac{12\pi^2 E}{23\,(l_e/r_g)^2}$$

Inelastic Buckling $$\sigma_{all} = \frac{\left\{1 - \left[\frac{(l_e/r_g)^2}{2C_c^2}\right]\right\} S_y}{n_\sigma}$$

Allowable Stress Reduction $$n_\sigma = \frac{5}{3} + \frac{3\,(l_e/r_g)}{8C_c} - \frac{(l_e/r_g)^3}{8C_c^3}$$

Example 9.3

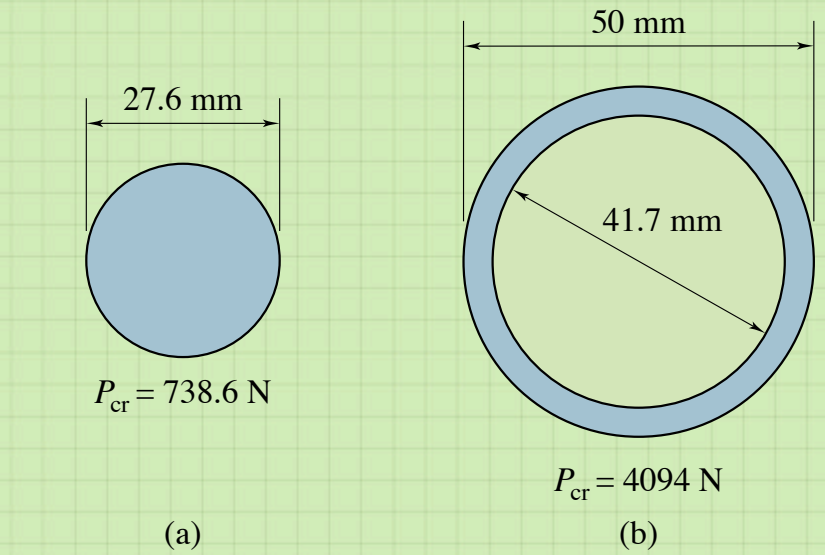
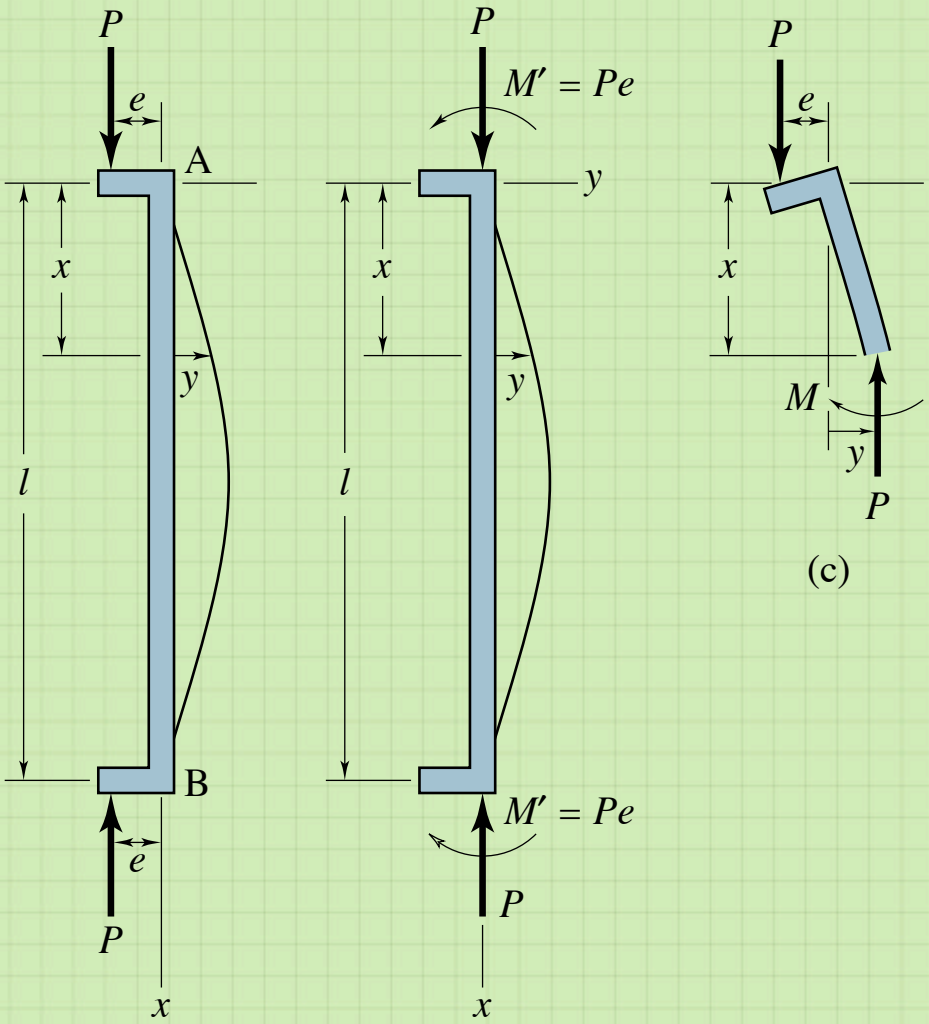


Figure 9.6 Cross-sectional areas, drawn to scale, from results of Exampe 9.3, aas well as critical buckling load for each cross-sectional area.

Eccentrically Loaded Column



Secant Equation:

$$y_{max} = e\left[\sec\left(\frac{l_e}{2}\sqrt{\frac{P}{EI}}\right) - 1\right]$$

Figure 9.7 Eccentrically loaded column. (a) Eccentricity; (b) statically equivalent bending moment; (c) free-body diagram through arbitrary section.